# Math 115
# Spring 2017
# Lecture 16

Exponential Rules:

① $x^n = \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$ , ② $x^1 = x$

③ $x^0 = 1, \; x \neq 0$ , ④ $x^m \cdot x^n = x^{m+n}$ ⑤ $(x^m)^n = x^{m \cdot n}$

⑥ $(xy)^n = x^n y^n$ ⑦ $\frac{x^m}{x^n} = x^{m-n}$ ⑧ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

⑨ $x^{-n} = \frac{1}{x^n}$ ⑩ $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$

⑪ $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

Simplify

1) $(x^3)^5 \cdot x^2$

$= x^{15} \cdot x^2 = \boxed{x^{17}}$

2) $\dfrac{(x^5)^4}{x^7} = \dfrac{x^{20}}{x^7} = \boxed{x^{13}}$

3) $(-3x^6)^3$

$= (-3)^3 (x^6)^3$

$= \boxed{-27x^{18}}$

4) $\dfrac{x^{-2} y^{10}}{x^{13} y^{-4}}$

$= \dfrac{y^4 \, y^{10}}{x^{13} \, x^2}$

$= \boxed{\dfrac{y^{14}}{x^{15}}}$

Distribute & Simplify

1) $4(3x^2 + 5x - 2) - 3(4x^2 + 7x - 3)$

$= 12x^2 + 20x - 8 - 12x^2 - 21x + 9$

$= -1x + 1 = \boxed{-x + 1}$

2) $x^2(3x - 5) - 4x(x^2 + 5x - 1)$

$= 3x^3 - 5x^2 - 4x^3 - 20x^2 + 4x$

$= \boxed{-x^3 - 25x^2 + 4x}$

3) $5x^3y^2(4x^2y^3 - 10xy + 1)$

$= \boxed{20x^5y^5 - 50x^4y^3 + 5x^3y^2}$

$x^3 \cdot x^2 = x^{3+2} = x^5$

## Use FOIL To multiply:

① $(x+2)(x+5)$

$= x^2 + 5x + 2x + 10$

$= \boxed{x^2 + 7x + 10}$

② $(2x+3)(x-4)$

$= 2x^2 - 8x + 3x - 12$

$= \boxed{2x^2 - 5x - 12}$

③ $(5x-2)(3x-4)$

$= 15x^2 - 20x - 6x + 8$

$= \boxed{15^2 - 26x + 8}$

④ $(3x+4)(3x-4)$

$= 9x^2 - 12x + 12x - 16$

$= \boxed{9x^2 - 16}$

⑤ $(x+2)(x^2 - 2x + 4)$

$= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8$

$= \boxed{x^3 + 8}$

## Special Products:

$(A+B)^2 = A^2 + 2AB + B^2$

$(x+7)^2 = x^2 + 2 \cdot x \cdot 7 + 7^2$

$= \boxed{x^2 + 14x + 49}$

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$(3x+5)^2 = (3x)^2 + 2 \cdot (3x)(5) + (5)^2$

$= \boxed{9x^2 + 30x + 25}$

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$(4x^3 + 7x^2)^2 = (4x^3)^2 + 2(4x^3)(7x^2) + (7x^2)^2$

$= \boxed{16x^6 + 56x^5 + 49x^4}$

$(A-B)^2 = A^2 - 2AB + B^2$

$(x-3)^2 = x^2 - 2 \cdot x \cdot 3 + 3^2$

$= \boxed{x^2 - 6x + 9}$

$(2x-5)^2 = (2x)^2 - 2(2x)(5) + (5)^2$

$= \boxed{4x^2 - 20x + 25}$

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$(3x^2 - 4y^3)^2 = (3x^2)^2 - 2(3x^2)(4y^3) + (4y^3)^2$

$= \boxed{9x^4 - 24x^2y^3 + 16y^6}$

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$(A+B)(A-B) = A^2 - B^2$

Conjugates

$(5x+3)(5x-3) = (5x)^2 - (3)^2 = \boxed{25x^2 - 9}$

$(8x-10)(8x+10) = (8x)^2 - (10)^2 = \boxed{64x^2 - 100}$

$(4x^2+9)(4x^2-9) = (4x^2)^2 - (9)^2$

$= \boxed{16x^4 - 81}$

Find the area & perimeter

$P = 2L + 2W$

$A = LW$

$5x^3 - 4$

$5x^3 + 4$

$P = 2L + 2W$

$= 2(5x^3 + 4) + 2(5x^3 - 4)$

$= 10x^3 + 8 + 10x^3 - 8$

$= \boxed{20x^3}$

$A = LW$

$= (5x^3 + 4)(5x^3 - 4)$

Conjugates

$= (5x^3)^2 - (4)^2 = \boxed{25x^6 - 16}$

Find A & P:

$A = S^2$

$P = 4S$

$10x^4 + 3x^2$

$10x^4 + 3x^2$

$A = (10x^4 + 3x^2)^2$

$= (10x^4)^2 + 2(10x^4)(3x^2) + (3x^2)^2$

$\boxed{A = 100x^8 + 60x^6 + 9x^4}$

$P = 4(10x^4 + 3x^2)$

$= \boxed{40x^4 + 12x^2}$

Simplify

① $\frac{30x^{12}y^8}{5x^3y^6} = 6x^9y^2$

② $\frac{-24x^{18}y^3}{4x^{10}y^{10}} = \frac{-6x^8}{y^7}$

$\frac{-35x^{14}y^{-3}}{-5x^{-6}y^{12}} = \frac{7x^{14}x^6}{y^3y^{12}} = \frac{7x^{20}}{y^{15}}$

Use $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$ to Simplify

① $\left(\frac{3x}{4y}\right)^{-2} = \left(\frac{4y}{3x}\right)^2 = \frac{16y^2}{9x^2}$

② $\left(\frac{5x^3}{y^{10}}\right)^{-3} = \left(\frac{y^{10}}{5x^3}\right)^3 = \frac{y^{30}}{125x^9}$

Simplify $\left(\frac{-2x^{-4}}{3y^{-8}}\right)^{-4}$

Hint: Make sure exponents inside ( ) are positive first.

$$= \left(\frac{-2y^{8}}{3x^{4}}\right)^{-4} = \left(\frac{3x^{4}}{-2y^{8}}\right)^{4} = \frac{3^{4}(x^{4})^{4}}{(-2)^{4}(y^{8})^{4}} = \boxed{\frac{81x^{16}}{16y^{32}}}$$

Scientific Notation

$N \times 10^{n}$ → can be any integer

$1 \le N < 10$

① $2.5 \times 10^{-17}$ — Indicates small number

.00000000000000025

17 times

② $6.75 \times 10^{32}$ — indicates large number

675000000000...0

32 times

Simplify

$$\frac{(1.5\times10^{12})\cdot(2.8\times10^{7})}{(8.3\times10^{-15})\cdot(7.5\times10^{-10})}$$

$= 4.2\times10^{12+7} = \boxed{4.2\times10^{19}}$

$= 62.25\times10^{-25}$

$= 6.225\times10^{1}\times10^{-25} = \boxed{6.225\times10^{-24}}$

$$(9.4\times10^{-17})\cdot(8.5\times10^{-14})$$

$= 79.9\times10^{-17+(-14)}$

$= 79.9\times10^{-31} = 7.99\times10^{1}\times10^{-31}$

$= 7.99\times10^{1+(-31)} = \boxed{7.99\times10^{-30}}$

$$\frac{4.8\times10^{25}}{2.5\times10^{10}} = 1.92\times10^{25-10} = \boxed{1.92\times10^{15}}$$

Simplify

$$\frac{1.5 \times 10^{-18}}{6 \times 10^{12}}$$

$= .25 \times 10^{-18-12} = .25 \times 10^{-30}$

$= 2.5 \times 10^{-1} \times 10^{-30}$

$= \boxed{2.5 \times 10^{-31}}$

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$$\frac{4.25 \times 10^{23}}{8.5 \times 10^{-12}}$$

$= .5 \times 10^{23-(-12)}$

$= \boxed{.5} \times 10^{35} = 5 \times 10^{-1} \times 10^{35} = \boxed{5 \times 10^{34}}$

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Monomial → Number · Variable(s) (whole #, exp.)

ex: $6x^3$, $-7x^2y^4$, $\frac{2}{3}x^3y^2z$, $1000$

$6x^3$ — Degree $= 3$

Coef. $= 6$

$-12x^4y^5$

Coef. $= -12$

Degree $= 4+5 = 9$

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$\frac{2}{3}x^6y^3z$ Coef. $\frac{2}{3}$

Deg. $= 6+3+1 = 10$

Binomial → Two monomials separated by + or −.

$4x+7$, $2x^2-8x$, $x^3-2y^3$, $1000+7x^4$

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$25x^3 - 100x^2$

| Monomial | Coef. | Degree |
|---|---|---|
| $25x^3$ | 25 | 3 |
| $-100x^2$ | −100 | 2 |

For the binomial

Deg. $= 3$

Leading Coef. $= 25$

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$10x^3y^8 + 2x^7y^6$

| Monomial | Deg. | Coef |
|---|---|---|
| $10x^3y^8$ | 11 | 10 |
| $2x^7y^6$ | 13 | 2 |

For the binomial

Deg. $= 13$

L.C. $= 2$

$4x^6y^8z^4 - 250xyz$

| Monomial | Coef. | Degree |
|---|---|---|
| $4x^6y^8z^4$ | 4 | $6+8+4=18$ |
| $-250xyz$ | $-250$ | $1+1+1=3$ |

For Bino.
D = 18
L.C. = 4

Constant is a monomial without Variable. Deg. = 0.

2017, Deg. = 0

Trinomial → when 3 monomials are Separated by + or −.

$4x^2 - 15x + 17$

For the Trinomial
D = 2
L.C. = 4

| Monomials | Coef. | Deg. |
|---|---|---|
| $4x^2$ | 4 | 2 |
| $-15x$ | $-15$ | 1 |
| 17 | Constant | 0 |

$20x^4y^3 - 100x^2y^8 + 2017$

D = 10, L.C. = −100

| Monomial | Coef. | Deg. |
|---|---|---|
| $20x^4y^3$ | 20 | 7 |
| $-100x^2y^8$ | −100 | 10 |
| 2017 | Const. | 0 |

Polynomial : When Monomials are Separated by + or −.

Monomial → Polynomial with 1 term.

Binomial → " " 2 terms.

Trinomial → " " 3 terms

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$18x^{12}y^{10}$ $-44x^{8}y^{10}$ $+100xy$ $-4000$

Coef. = 18 Coef. = −44 Coef. = 100 Constant

D = 22 D = 18 D = 2 D = 0

Degree = 22, LC = 18

Dividing Polynomial by Monomial:

$$\frac{28x^3 + 14x^2 + 7x}{7x}$$

$$= \frac{28x^3}{7x} + \frac{14x^2}{7x} + \frac{7x}{7x}$$

$$= \boxed{4x^2 + 2x + 1}$$

Divide

$$\frac{35x^4 - 25x^2 + 5x}{-5x}$$

$$= \frac{35x^4}{-5x} - \frac{25x^2}{-5x} + \frac{5x}{-5x}$$

$$= \boxed{-7x^3 + 5x - 1}$$

$$\frac{24x^6}{4x^2} - \frac{12x^4}{4x^2} + \frac{16x^2}{4x^2}$$

$$= 6x^4 - 3x^2 + 4$$

Deg. 4
LC = 6, Const = 4

$$\frac{24x^6 - 12x^4 + 16x^2}{4x^2} =$$

Simplify

$$\frac{10x^4y^3 - 20x^2y + 30xy^2}{10x^2y^2}$$

SG 12
Due
Tuesday

$$= \frac{10x^4y^3}{10x^2y^2} - \frac{20x^2y}{10x^2y^2} + \frac{30xy^2}{10x^2y^2}$$

$$= x^2y - \frac{2}{y} + \frac{3}{x}$$

Not a Polynomial because variable left in the denom.